



## Product Codes for Optical Communication

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# PRODUCT CODES FOR OPTICAL COMMUNICATION

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**Abstract** Many optical communication systems might benefit from forward-error-correction. We present a hard-decision decoding algorithm for the "Block Turbo Codes", suitable for optical communication, which makes this coding-scheme an alternative to Reed-Solomon codes.

## Introduction

Forward-error-correction is a highly developed method for improving communication on radio-frequency links. Many optical communication systems might benefit from this technology also. However, the requirements for the coding schemes are different on several points. One thing is the extreme data rates used with optical communication – requiring very fast error-correcting decoders. Another important thing is the lack of soft-decision. In radio communication the demodulated symbols often appear with some reliability - known as soft-decision – and the coding systems are designed for this, giving an improvement of several dB. This reliability information is not available, or very restricted, in optical systems. Therefore the coding systems must be designed for hard-decision.

Reed-Solomon codes are very powerful codes with a well-structured and fast decoding algorithm for hard-decision. Consequently the (255,239,17) Reed-Solomon code, which is capable of correcting 8 byte errors in a word of 255 bytes (of which 239 is information), has been used as a de facto standard for optical communication.

A system that might compete with the Reed-Solomon codes is product codes based on Hamming (or BCH) codes. One such system is the "Block Turbo Codes" [1], which is based on an iterative decoding algorithm with soft-decision. In this paper we will develop a hard-decision decoding algorithm for these product codes. We will show that this decoding algorithm is able to correct all error-patterns of weight less than half the minimum weight of the code, and compare the simulated performance with the results from [1].

## The Product Codes

The concept of product codes is to construct very long block codes based on short constituent codes. The resulting code is not the best possible for the given dimensions, but since the code is composed of smaller block codes the decoding can be less complicated.

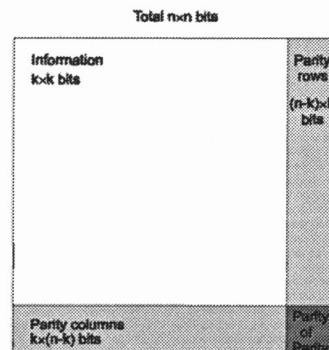
In this paper we use two constituent codes:

The (63,57,3) Hamming code. Since the minimum distance is 3 this code can correct all single errors.

The (64,57,4) extended Hamming code. This is just the (63,57,3) code with an additional parity bit calculated as the modulo-2 sum of the original codeword. This code corrects single errors and detects double errors.

A two-dimensional product code is constructed as follows:

$k \times k$  information bits are aligned in a square. Each row is encoded with the  $(n,k,d_{\min})$  constituent code, giving  $k$  new columns. Each column, including the new parity columns, are encoded giving  $k$  new rows of  $n$  bit. Since the constituent code is linear these new rows will also be codewords. The resulting encoded block is an  $n \times n$  square where each row and each column is a codeword. The minimum weight of the product code is  $d_{\min} \times d_{\min}$ .



## Product code.

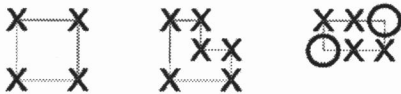
With the (63,57,3) code we get a (3969,3249,9) product code with rate  $R=0.82$ . With the (64,57,4) code a (4069,3249,16) product code with  $R=0.79$ .

## The Decoding Algorithm

The main principle of the decoding algorithm is to make iterative decoding in the two dimensions. The algorithm does not use true soft-decision but some of the bits are marked as unreliable by the constituent decoders. The iterations are slightly different for the two constituent codes.

For (64,57,4) the constituent decoder regards the marked positions from the other dimension as errors, and calculates the syndrome. If the syndrome is 0, i.e. the input is a codeword, the marked positions are corrected. If the syndrome indicates one error and this position is not marked a new mark is set to be used in the succeeding decoding in the other dimension. For (63,57,3) an initial syndrome is calculated without regard to the marked positions. If this syndrome is zero no further actions are taken, otherwise the constituent decoder continues as described for the (64,57,4) code.

The error patterns not corrected by these iterations are closed chains of errors or false marks as illustrated in the figure.



Closed chains of errors and false marks. X=error, O=marked from row decoder (with (63,57,3)).

To correct these error patterns we have developed three "clean-up" routines, to be applied between the iterations.

Clean-up A marks the rows with a non-zero syndrome or marked positions from the column decoders. If no subset of these rows form a codeword corrections are attempted in combinations of these rows. If a combination gives syndrome=0, this combination of bits are corrected. If a subset of the marked rows form a codeword a similar procedure is applied to the columns. Clean-up B is identical to Clean-up A, except that only rows with non-zero syndromes are marked. For Clean-up C each constituent decoder corrects the marked positions from the other dimension, calculates the syndrome and marks a new position if the syndrome is non-zero.

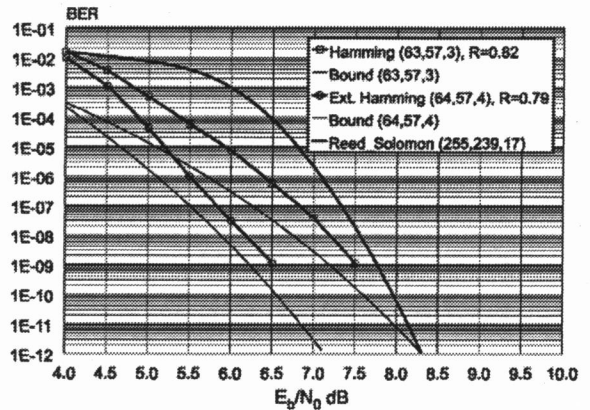
### Simulation Results

The performance of the coding system has been simulated. We have used 30 iterations with Clean-up A after each iteration, Clean-up B after iteration 10 and 30 and Clean-up C after iteration 20. The results are shown in the figure. The number of iterations (30) used may be reduced significantly with minor or no loss, but is used here to ensure the best possible performance. The performance is shown as Bit-Error-Rate after decoding vs.  $E_b/N_0$ , where  $E_b$  is the energy per information bit and  $N_0$  the noise spectral density. For the (64,57,4) code 6.5 dB corresponds to an error probability on the channel of  $3.9E-03$ . For the

(63,57,3) code 7.5 dB corresponds to  $1.2E-03$ .

We have calculated bounds to the performance based only on the minimum weight words. The performance of the coding scheme can never be better than this bound, but will approach it for high signal-to noise ratios.

Finally the performance of the (255,239,17) Reed-Solomon code is calculated and shown in the figure.



Simulated results compared to performance of Reed-Solomon (255,239,17).

### Conclusions

Compared to the soft-decision results in [1], we loose about 2 dB, which is close to the expected gain from soft-decision from the channel. We can conclude that the iterative "turbo" principle works for hard-decision. Also, the complexity of the hard-decision decoder is significantly less than the complexity of the soft-decision decoder, which implies 10-15 times the number of Hamming code decodings and a complex distance measure on the results.

The Performance of the product code is better than the performance of the (255,239,17) Reed-Solomon code at low signal-to-noise ratios, and for the extended Hamming code also at high SNR's. However, this is on the expense of a lower rate and a longer block size. Estimation of the actual complexity of the decoder requires further study, but we believe that it will be possible to implement a fast an effective decoder for this decoding algorithm.

### References

- 1 R. M. Pyndiah, "Near Optimum Decoding of Product Codes: Block Turbo Codes", IEEE Trans. Communication, vol. 46, Aug 1998, pp. 1003-1010.